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Equalization Payments in a Bargaining Model of Tax Competition

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Abstract

A model in which a high-productivity region and a low-productivity region bargain with each firm in a group of mobile firms is constructed. It differs from the Han and Leach [7] model in that the firms are identical, so that its comparative statics are more tractable. The model is used to examine the allocative effects of equalization payments (both non-contingent payments and “corrective subsidies”). The equilibrium is characterized by misallocation of capital and underprovision of public goods. Underprovision is more severe in the low-productivity region than the high-productivity region. A transfer of revenue from the high-productivity region to the low-productivity region augments public goods provision in the low-productivity region, allowing that region to make more generous offers to the firms. Likewise, underprovision becomes more severe in the high-productivity region, so that its offers become less generous. Equilibrium is attained by a movement of firms from the high-productivity region to the low-productivity region, reducing the misallocation of capital.

*I am indebted to Seungjin Han for valuable discussions.

1. Introduction

A mobile firm's location decision is influenced by the rates at which its profits would be taxed in the various locations open to it. The resource and welfare implications of the strategic setting of these tax rates by competing governments has been extensively studied (see Wilson [11] for a survey of the literature). However, the firm's location decision is also influenced by other financial incentives, such as tax holidays and infrastructure grants.¹ Since these incentives can be tailored to individual firms while tax rates cannot, they are likely to have quite different effects on resource allocation. The literature on financial incentives is small, and almost all of it employs a partial equilibrium framework. Tax holidays have received the most attention. A number of earlier papers have offered explanations of the tax breaks given to mobile firms, but these papers have described the negotiations between a single firm and one or two governments. Doyle and van Wijnbergen [6] examine the intertemporal structure of a firm's tax payments. They note that a mobile firm has greater bargaining power than a firm that has already incurred the sunk costs associated with locating in a particular region. They argue that mobile firms will use their extra bargaining power to extract concessions. Bond and Samuelson [3] present an alternative explanation of the same phenomenon: a region can offer a tax holiday to a mobile firm to signal that firms that locate there experience high productivity. The firm will willingly pay higher tax rates in later periods because it is very productive, and these high tax rates allow the government to recover the cost of the initial tax holiday. A low-productivity region could not offer the same incentive: firms that located there would relocate when they found that they had low productivity, so the region would be unable to recover the cost of the tax holiday. King, McAfee and Welling [8] extend this model by allowing the firm to negotiate simultaneously with two governments, and add a stochastic element to the regional productivities. Black and Hoyt [1] take an altogether different approach, arguing that subsidies to mobile firms can undo the distortionary effects of average cost pricing of publicly provided services.

These models have partial equilibrium structures, so they cannot describe the impact of financial incentives on resource allocation and welfare. A general equilibrium model is required for this purpose. Seungjin Han and I [7] have

¹As one example of the importance of financial incentives, Ford is currently (January 2008) seeking \$60 million from the Ontario and Canadian governments to assist it in reopening an engine plant. The cost of refitting the plant is approximately \$300 million.

proposed one such model, and I am aware of no others. Our model assumes that the government of each region makes firm-specific offers to every firm, and that each firm chooses to locate in the region in which its after-tax profits are maximized. We show that the equilibrium allocation is Pareto optimal if the government is also able to impose a lump-sum tax on its citizens (as is commonly assumed in the fixed-rate tax competition literature), but that misallocation of capital and underprovision of the public good are possible—though not certain—if the government cannot impose such a tax. The assumption that the government is willing to “cut a deal” with every firm is somewhat unrealistic, but we would argue that the standard tax competition model (which assumes that there are no deals for any firm) is also unrealistic. Our model and the fixed-rate model constitute polar cases, with reality somewhere in between.

Our model assumes that each firm’s productivity varies from region to region, and that the profile of regional productivities varies from firm to firm. The heterogeneity of the firms plays a crucial role in establishing the equilibrium, so the distribution of these profiles must be non-degenerate. Unfortunately, the resulting model is sufficiently complex that its comparative statics can be intractable.

The current paper presents a more tractable version of the Han and Leach [7] model. The greater tractability is obtained by assuming that the firms are homogeneous. Following Boadway and Flatters [2], the model assumes that there are two regions, and that all firms have high productivity in one region and low productivity in the other. Wilson’s [11] assumption that there is only one tax, the profits tax, is imposed. The equilibrium is characterized by misallocation of capital (too many firms locate in the high productivity region) and underprovision of public goods. Although each region underprovides the public good, the degree of underprovision is more severe in the low productivity region. These two results are related. As the underprovision of public goods becomes more severe, the share of gross profits taken by the government becomes larger. Equilibrium requires the firms to be indifferent between the regions; equivalently, it requires that each firm’s after-tax profits are the same in both regions. This condition is met only if gross profits are bigger in the region in which the government takes the larger share of gross profits. Hence, gross profits must be lower in the high productivity region than in the low productivity region, signalling a misallocation of capital.

The new model is applied to the issue of equalization payments. The relationship between equalization payments and tax competition has been extensively studied in the context of the standard tax competition model, notably by Kothenburger [9] and by Bucovetsky and Smart [4]. The intent of the current paper is

to discover whether the beneficial properties of these systems carry over to the bargaining model. Equalization payments are modelled as a transfer (instituted by some higher authority) of tax revenue from the low productivity region to the high productivity region. The channel through which equalization payments operate is clear: they reduce the severity of underprovision in the low productivity region and raise it in the high productivity region, diminishing the gap between the shares of gross profits taken by the two governments. The gap between gross profits of firms in the two regions is likewise diminished, indicating a smaller degree of resource misallocation. While these effects serve to reduce misallocation, non-contingent equalization payments cannot induce a Pareto optimal allocation. That is, there is misallocation of capital and underprovision of public goods even in the presence of a non-contingent equalization payment. It is sometimes, but not always, possible to reach a Pareto optimal allocation through “corrective subsidies” of the sort described by Wildasin [10] and DePater and Myers [5]. These subsidies sometimes fail because there is a definite upper limit on the amount of revenue that they can raise; if preferences for the public good are sufficiently strong, they cannot raise enough revenue to provide the requisite public goods.

2. Production and Welfare

The economy consists of two regions, denoted H and L , and each region is populated by a continuum of identical agents. The agents in each region have measure 1, and each agent inelastically offers one unit of labour. There is also a continuum of identical firms. The continuum has measure 1 and the firms are indexed by the numbers on the unit interval. The firms locate in one of the two regions, and produce a homogeneous good using the labour available in that region.

Let σ_i ($i = H, L$) be the set of firms that locate in region i , and let s_i be its measure. Let the map $n : [0, 1] \rightarrow \mathcal{R}_+$ specify the quantity of labour employed by each firm. The output of firm j located in region i is

$$y_i(j) = \theta_i n(j)^\alpha \quad 0 < \alpha < 1$$

Here, θ_i is a regional productivity factor. Region H is the high productivity region and region L is the low productivity region:

$$\theta_H = 1$$

$$\theta_L = \lambda \quad 0 < \lambda < 1$$

Aggregate output in region i is

$$Y_i = \int_{\sigma_i} y_i(j) dj = \theta_i \int_{\sigma_i} n(j)^\alpha dj \quad (1)$$

If the firms in region i share the available labour equally, firm j 's employment of labour is

$$n(j) = \frac{1}{s_i} \quad \text{for all } j \in \sigma_i \quad (2)$$

and region i 's aggregate output is

$$Y_i = \theta_i (s_i)^{1-\alpha} \quad (3)$$

One unit of output can be converted into either one unit of private goods or one unit of public goods, so the economy's production possibility frontier is

$$Y_L + Y_H \geq c_L + c_H + g_L + g_H \quad (4)$$

where c_i and g_i are region i 's consumption of the private and public goods respectively.

Since the agents in each region are identical and have measure one, each agent's consumption bundle is (c_i, g_i) . Each agent's utility is a strictly increasing function of his consumption:

$$U_i = u(c_i, g_i)$$

It is assumed that u is strictly concave and class C^2 . It is also assumed that both goods are normal, and that the income elasticity of demand for the public good is no greater than 1. Let MRS be the value of a unit of public goods measured in private goods:

$$MRS(c_i, g_i) \equiv \frac{u_2(c_i, g_i)}{u_1(c_i, g_i)}$$

The assumptions on income elasticities imply that²

$$MRS(c_i, g_i) > MRS(ac_i, bg_i) \quad \text{if } b > 1 \text{ and } b > a \quad (5)$$

An *allocation* in this economy is a collection $(\sigma_L, \sigma_H, n, c_L, c_H, g_L, g_H)$. An allocation is *feasible* if

²Draw the income expansion paths in the (c_i, g_i) plane. Then (bc_i, bg_i) does not lie on a lower expansion path than (c_i, g_i) , and (ac_i, bg_i) lies on a higher expansion path than (bc_i, bg_i) .

1. It satisfies (1) and (4), so that total consumption is no greater than total production.
2. Every firm locates in one of the two regions:

$$\sigma_L \cap \sigma_H = \emptyset$$

$$\sigma_L \cup \sigma_H = [0, 1]$$

3. The firms in each region use only the labour available within the region:

$$\int_{\sigma_i} n(j) dj \leq 1 \quad i = H, L$$

Note that this definition of feasibility assumes that goods produced in one region can be consumed in either region.

3. Pareto Optimality

An allocation is Pareto optimal if there is no other allocation under which utility is higher in at least one region and lower in neither region. Since the production possibility frontier allows goods produced in each region to be consumed in either region, the conditions for Pareto optimality can be broken into two parts: the maximization of output, and the allocation of output.

Suppose that the firms have been divided between the two regions in some fashion. Since the firms are identical, and since the marginal product of labour is diminishing, output is maximized only if every firm in a region employs the same quantity of labour. As well, no labour can be left idle. These conditions imply that the allocation of labour across firms satisfies (2). Total output is then

$$Y \equiv Y_H + Y_L = \theta_H(s_H)^{1-\alpha} + \theta_L(s_L)^{1-\alpha}$$

Output is maximized if moving an arbitrarily small measure of firms from one region to the other does not raise output:

$$\theta_H(s_H)^{-\alpha} = \theta_L(s_L)^{-\alpha} \tag{6}$$

Any feasible (σ_L, σ_H, n) that satisfies (2) and (6) maximizes total output.

Now suppose that output has been maximized. Since utility is strictly increasing in consumption of the two goods, every unit of output must be allocated to one of the two regions:

$$Y_L + Y_H = c_L + c_H + g_L + g_H \quad (7)$$

Given the output available to region i , each agent's utility is maximized if

$$u_1(c_i, g_i) = u_2(c_i, g_i) \quad (8)$$

If (c_H, c_L, g_H, g_L) satisfies (7) and (8) for both i , any reallocation of output reduces utility in at least one region.

An allocation is Pareto optimal if and only if it satisfies the output maximization conditions (2) and (6) and the output allocation conditions (7) and (8).

4. Tax Competition

This section sets out a variant of the Han and Leach [7] bargaining model of tax competition.³ Each region's public good is provided by its government, which has three fiscal instruments at its disposal. It can offer financial incentives to specific firms to induce them to locate within the region, and it can tax the profits of the firms that choose to locate there. Since the government taxes profits at a fixed rate but is able to offer financial incentives that vary from firm to firm, identical firms in the same region do not necessarily have the same net profits. The last fiscal instrument is a non-negative lump-sum transfer to the citizens, enabling them to augment their private consumption.⁴ The government's tax revenue must equal its expenditures, which are the sum of the financial incentives, the lump-sum transfer and spending on the public good.

The model is a two stage game. In the first stage, each government makes an offer to each firm, and each firm chooses to locate in the region (or one of the regions) in which its net profits is maximized. In the second stage, the firms hire labour in competitive regional markets, produce goods, receive financial incentives and pay taxes, and return their net profits to their owners. The ownership of each

³A key element of that model was heterogeneity across firms. An interesting aspect of the current model is that it generates many of the same results without assuming any kind of heterogeneity.

⁴The non-negativity of the transfer implies that there is no lump-sum tax in the economy. It will be shown below that the optimal transfer is zero.

firm is equally divided among all of the agents, so both regions receive an equal share of net profits. Each government chooses the division of its revenue between public goods provision and the lump-sum transfer. The firms and the governments correctly anticipate the outcome of the second stage when they make their choices in the first stage.

A government's offer to a firm states the *maximal net profits* that the firm would earn if it located within the region. These profits are defined as the net profits earned when the firm hires the optimal quantity of labour at the competitive wage rate. A government can make such offers because it knows the other government's offers, and therefore can anticipate the market-clearing wage and the division of labour across firms. Each firm locates in the region in which its maximal net profits are higher; if both regions offer the same maximal net profits, the firm is willing to locate in either region. Each government, taking the offers of the other government as given and correctly anticipating the outcome of the second stage, chooses the offers that maximize the utility of the region's citizens.

Let $\pi_i : [0, 1] \rightarrow \mathcal{R}_+$ be the maximal net profits offered to each firm by the government of region i . The outcome of the game is described by a profile (π_H, π_L) and an allocation $(\sigma_L, \sigma_H, n, c_L, c_H, g_L, g_H)$.

Consider the second stage first. The firms in each region are identical and hire labour at the market-clearing wage rate, so each firm hires the same quantity of labour. Aggregate output in region i is (3). Government revenue, defined as tax revenue less financial incentives, is equal to the difference between gross profits and maximal net profits.

$$R_i = (1 - \alpha)\theta_i(s_i)^{1-\alpha} - \int_{\sigma_i} \pi_i(j) dj \quad i = H, L \quad (9)$$

The total income of the region's citizens is the sum of their wage earnings (which are equal to αY_i under competition) and half of the profits of every firm:

$$X_i = \alpha\theta_i(s_i)^{1-\alpha} + \frac{1}{2} \left[\int_{\sigma_H} \pi_H(j) dj + \int_{\sigma_L} \pi_L(j) dj \right] \quad i = H, L \quad (10)$$

Letting t be the transfer to the citizens, the government's maximization problem is

$$\max_{t \geq 0} U_i = u(X_i + t, R_i - t)$$

Its solution is described by the Kuhn-Tucker conditions

$$t \geq 0$$

$$\begin{aligned}
u_1(c_i, g_i) &\leq u_2(c_i, g_i) \\
t[u_1(c_i, g_i) - u_2(c_i, g_i)] &= 0
\end{aligned} \tag{11}$$

Public goods are *optimally provided* in region i if

$$u_1(c_i, g_i) = u_2(c_i, g_i)$$

and they are *underprovided* if

$$u_1(c_i, g_i) < u_2(c_i, g_i)$$

Clearly, optimal provision occurs only when government revenue R_i is a sufficiently large part of total resources $X_i + R_i$.

Now consider the first stage. Let $-i$ denote the region that is not i , and define the sets

$$\begin{aligned}
\underline{\sigma}_i(\pi_H, \pi_L) &= \{j \in [0, 1] : \pi_i(j) > \pi_{-i}(j)\} \\
\overline{\sigma}_i(\pi_H, \pi_L) &= \{j \in [0, 1] : \pi_i(j) \geq \pi_{-i}(j)\}
\end{aligned}$$

That is, $\underline{\sigma}_i$ and $\overline{\sigma}_i$ are the smallest and largest sets of firms that could locate in region i under any profile (π_H, π_L) . The sets σ_H and σ_L are consistent with net profit maximization—henceforth, *consistent*—if they are feasible and satisfy the condition

$$\underline{\sigma}_i(\pi_H, \pi_L) \subseteq \sigma_i \subseteq \overline{\sigma}_i(\pi_H, \pi_L) \quad i = H, L$$

The choices made by the governments and firms in the first stage can now be more formally described.

Definition 1. Assume that each region anticipates that government revenue and total income will be given by (9) and (10), and that it will choose the optimal transfer in the second stage. An equilibrium includes a profile (π_H, π_L) and a consistent division of the firms (σ_H, σ_L) such that:

1. There is no alternative profile (π'_H, π'_L) , and no alternative division of the firms (σ'_H, σ'_L) that is consistent under this profile, such that U_H is higher.
2. There is no alternative profile (π_H, π'_L) , and no alternative division of the firms (σ'_H, σ'_L) that is consistent under this profile, such that U_L is higher.

An equilibrium profile has the following properties.

Lemma 2. *Let (π_H, π_L) be an equilibrium profile. Then, for each region i ,*

1. *The set $\{j \in [0, 1] : \pi_i(j) > \pi_{-i}(j)\}$ has measure zero.*
2. *There is no positive number k_i such that the sets $\{j \in \sigma_i : \pi_i(j) > k_i\}$ and $\{j \in \sigma_{-i} : \pi_i(j) < k_i\}$ both have positive measure.*

These properties imply that, in equilibrium, each region makes the same offer π to almost every firm. Every firm is indifferent between the two regions, and consistency requires only that every firm ends up in one of the two regions:

$$s_H + s_L = 1 \tag{12}$$

A triplet (π, s_H, s_L) that corresponds to an equilibrium has the following property.

Lemma 3. *Assume that each government anticipates that government revenue and total income will be given by (9) and (10), and that it will choose the optimal transfer in the second stage. Let (π_H, π_L) be a profile satisfying*

$$\pi_H(j) = \pi_L(j) = \pi \quad \text{for all } j \in [0, 1]$$

Let (σ_H, σ_L) be a consistent division of the firms, and let (s_H, s_L) be the measures of these sets. The profile and the division of the firms constitute an equilibrium outcome of the first stage of the tax competition game if and only if there is no alternative pair (s'_H, s'_L) satisfying (12) such that at least one of the regions attains higher utility.

That is, if each region makes the same offer π to every firm, and if there is no redistribution of the firms that raises utility in either region, neither region has a utility-improving deviation of *any* kind. Moreover, if there is some reallocation of the firms that would raise some region's utility under the current offer profile, that region can engineer a utility-improving redistribution of the firms by unilaterally adjusting its offers.

Given this result, an equilibrium triplet (π, s_H, s_L) is easy to find. It has the property that each region i is content with the fraction s_i of the firms when the common offer is π .

Since every firm in region i receives the same offer and employs the same quantity of labour, total income and government revenue in region i are

$$\hat{X}(s_i, \pi; \theta_i) \equiv \alpha \theta_i (s_i)^{1-\alpha} + \frac{\pi}{2} \tag{13}$$

$$\widehat{R}(s_i, \pi; \theta_i) \equiv (1 - \alpha)\theta_i(s_i)^{1-\alpha} - s_i\pi \quad (14)$$

The functions \widehat{X} and \widehat{R} are class C^2 and concave in s_i . Since u has the same properties, the maximum value function

$$V(s_i, \pi; \theta_i) \equiv \max_{t \geq 0} u(\widehat{X}(s_i, \pi; \theta_i) + t, \widehat{R}(s_i, \pi; \theta_i) - t)$$

is also class C^2 and concave in s_i . Consequently, if each region's optimal share of the firms is in the interior of the interval $[0, 1]$, it is a stationary point:

$$V_1(s_i, \pi; \theta_i) = 0 \quad i = H, L \quad (15)$$

The concavity of V implies that the stationary point—if it exists—is unique.

Lemma 4. *In any equilibrium:*

1. *Some firms are located in each region.*
2. *Public goods are underprovided and the lump-sum transfer is 0 in both regions.*

Since the first stage equilibrium is not a corner solution, it is a triplet (s_H, s_L, π) satisfying (12) and (15). The continuity of V_i implies that the optimal value of s_i (call it s_i^*) is a continuous function of π and θ_i . The derivatives of this function are determined by the signs of V_{12} and V_{13} respectively. When evaluating these derivatives, the marginal rate of substitution can be represented by the composite function

$$\mu(s_i, \pi; \theta_i) \equiv MRS(\widehat{X}(s_i, \pi; \theta_i), \widehat{R}(s_i, \pi; \theta_i))$$

because, by Lemma 4, the optimal transfer is zero.

1. Differentiation of the first-order condition gives

$$\frac{\partial s_i^*(\pi, \theta_i)}{\partial \pi} = - \frac{V_{12}}{V_{11}} \Big|_{s_i^*}$$

V_{11} is negative (because V is concave in s_i), so the sign of this expression is the same as the sign of V_{12} evaluated at $s_i^*(\pi)$. Differentiating the equation

$$V_1(s_i, \pi; \theta_i) = u_1 \left[\widehat{X}_1(s_i, \pi; \theta_i) + \mu(s_i, \pi; \theta_i) \widehat{R}_1(s_i, \pi; \theta_i) \right]$$

gives

$$V_{12}|_{s_i^*} = u_1|_{s_i^*} \left(\widehat{X}_{12} + \widehat{R}_1 \mu_2 + \mu \widehat{R}_{12} \right) \Big|_{s_i^*} = u_1|_{s_i^*} \left(\widehat{R}_1 \mu_2 - \mu \right) \Big|_{s_i^*}$$

An increase in π increases \widehat{X} and reduces \widehat{R} , so μ_2 is positive. Furthermore, (15) implies

$$\left(\widehat{X}_1 + \mu \widehat{R}_1 \right) \Big|_{s_i^*} = 0 \quad (16)$$

and hence

$$\widehat{R}_1 \Big|_{s_i^*} < 0$$

It follows that V_{12} , evaluated at $s_i^*(\pi)$, is also negative and thus

$$\frac{\partial s_i^*}{\partial \pi} < 0$$

2. Differentiation of (15) gives

$$\frac{\partial s_i^*}{\partial \theta_i} = - \frac{V_{13}}{V_{11}} \Big|_{s_i^*}$$

where

$$V_{13}|_{s_i^*} = u_1|_{s_i^*} \left(\widehat{X}_{13} + \widehat{R}_1 \mu_3 + \mu \widehat{R}_{13} \right) \Big|_{s_i^*}$$

The derivatives \widehat{X}_{13} and \widehat{R}_{13} are both positive, and \widehat{R}_1 is again negative. An increase in θ_i raises X_i by a smaller proportion than it raises R_i , so μ_3 is negative. It follows that

$$\frac{\partial s_i^*}{\partial \theta_i} > 0$$

These results allow Figure 1 to be drawn. The intersection of the two curves determines the equilibrium values of s_H and π , and the equilibrium value of s_L is $1 - s_H$. The more productive region obtains more than half of the firms. The characteristics of the equilibrium are described by (16). Although each region gives up government revenue to attract the marginal firm, that firm raises the wage earnings of the region's citizens. Total wages are equal to the fraction α of total output. Since region H has both the higher productivity factor and the greater number of firms, it has the higher wage rate. Net profits are equally divided between the two regions, so region H also has the higher per capita income.

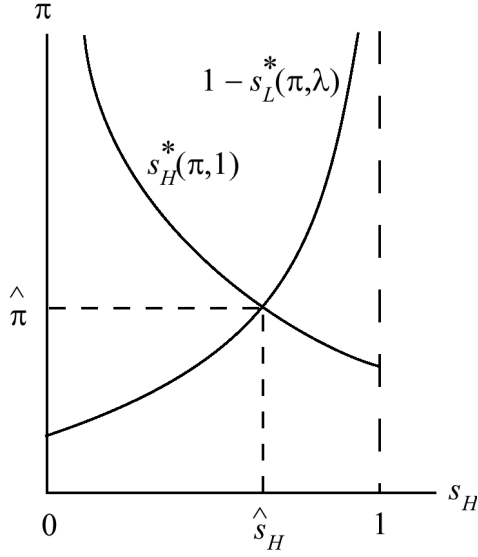


Figure 1: The equilibrium is $(\hat{s}_H, 1 - \hat{s}_H, \hat{\pi})$. More than half of the firms locate in the high productivity region.

The equilibrium is not Pareto optimal. Conditions (2) and (7) are satisfied, but (6) and (8) are violated. The violation of (8) has already been demonstrated; and the following lemma deals with the violation of (6) and its consequences.

Proposition 5. *In equilibrium,*

1. *The measure of the firms in the high productivity region is too large to maximize total output:*

$$\frac{\partial Y_H}{\partial s_H} < \frac{\partial Y_L}{\partial s_L}$$

2. *The marginal rate of substitution is higher in the low productivity region than in the high productivity region:*

$$\mu(s_L, \pi) > \mu(s_H, \pi) > 1$$

3. *Consumption of both goods is greater in the high productivity region than in the low productivity region.*

A region's ability to provide public goods largely determines its tax structure. By (15),

$$\pi = \frac{dY_i}{ds_i} \left[\frac{\alpha}{\mu(s_i, \pi)} + (1 - \alpha) \right] \quad (17)$$

The net profits of a firm in region i are π while its gross profits are dY_i/ds_i .^{5,6} If region i is somehow able to provide the optimal quantity of public goods (so that $\mu(s_i, \pi)$ is equal to 1), it allows the firm to retain all of the gross profits. As its ability to provide these goods declines (i.e. μ_i rises), the region becomes less generous, with the firm retaining a progressively smaller share of gross profits. In the equilibrium, the regions allow the same net profits, but for different reasons. A firm in region H is relatively unproductive; but the region contains the bulk of the firms so it is able to provide a relatively large quantity of public goods and hence allows the firm to retain a relatively large fraction of relatively small gross profits. The opposite situation prevails in region L : the firms receive a relatively small fraction of relatively large gross profits.

In summary, these are the properties of equilibrium:

1. Almost every firm receives an offer of $\hat{\pi}$ from both regions.
2. Any division of the firms between the regions is consistent with equilibrium as long as the measures of the firms going to regions H and L are \hat{s}_H and $1 - \hat{s}_H$ respectively. There are more firms in the high productivity region than in the low productivity region.
3. Every firm in region i ($i = H, L$) employs the same quantity of labour.
4. All of X_i is spent on the consumption good and all of R_i is spent on public goods. Consumption of both goods is greater in the high productivity region than in the low productivity region.

⁵The output of an individual firm in region i is $\theta_i(s_i)^{-\alpha}$. The firm's gross profits are the fraction $1 - \alpha$ of its output, which is also dY_i/ds_i .

⁶In the bargaining model with heterogeneous firms, the equilibrium is Pareto optimal if the governments have access to a lump-sum tax. The same result occurs here. Each government would use the lump-sum tax to optimally provide public goods, so the marginal rate of substitution would be the same in each region. Then (17) implies that dY_i/ds_i is the same in each region, so that output is maximized. If the government has access to both a profits tax and a lump-sum tax, the profits tax would raise no revenue when the firms are homogeneous but would raise positive revenue when the firms are heterogeneous.

5. Public goods are underprovided. The marginal rate of substitution is higher in the low productivity region than in the high productivity region.

5. Equalization Payments

Equalization payments in Canada are intended to ensure that every region is able to provide an adequate quantity of public goods. They are modelled here as a transfer of government revenue across regions, engineered by some central authority whose behaviour is exogenous. Assume that region i receives a transfer τ_i . Its revenue is no longer given by (14); instead, it is

$$\widehat{R}(s_i, \pi; \theta_i, \tau_i) \equiv (1 - \alpha)\theta_i(s_i)^{1-\alpha} - s_i\pi + \tau_i \quad (18)$$

Since the central authority is not invested with any additional taxing powers, the sum of the transfers must be zero. Region L provides the smaller quantity of public goods and has the larger marginal rate of substitution, so it will be taken to be the recipient of a positive transfer:

$$\tau_L = -\tau_H > 0$$

It is, of course, possible to give region L such a large transfer that it provides a larger quantity of public goods than region H and has a smaller marginal rate of substitution. It is assumed henceforward that the transfer is not so large as to generate this reversal of fortunes, and under this assumption, the adjustments to the model necessitated by equalization payments are quite small. Neither region is able to provide the optimal quantity of public goods, so the lump-sum transfer to the agents continues to be zero. Maximal utility is therefore

$$V(s_i, \pi; \theta_i, \tau_i) \equiv u(\widehat{X}(s_i, \pi; \theta_i), \widehat{R}(s_i, \pi; \theta_i, \tau_i))$$

and the marginal rate of substitution is

$$\mu(s_i, \pi; \theta_i, \tau_i) \equiv MRS(\widehat{X}(s_i, \pi; \theta_i), \widehat{R}(s_i, \pi; \theta_i, \tau_i))$$

The properties of equilibrium are essentially unchanged.

An equilibrium triplet (π, s_H, s_L) satisfies (12) and the first-order conditions

$$V_1(s_i, \pi; \theta_i, \tau_i) = 0 \quad i = H, L \quad (19)$$

To discover the effects of an increase in τ_L , let $s_i^*(\pi; \theta_i, \tau_i)$ satisfy (19). The signs of the first two partial derivatives are not changed by the introduction of equalization payments:

$$\frac{\partial s_i^*}{\partial \pi} < 0, \frac{\partial s_i^*}{\partial \theta_i} > 0$$

The new partial derivative is

$$\frac{\partial s_i^*}{\partial \tau_i} = - \frac{V_{14}}{V_{11}} \bigg|_{s_i^*} = - \frac{u_1 \left(\mu_4 \hat{R}_1 + \mu \right)}{V_{11}} \bigg|_{s_i^*} > 0$$

The positive sign of this derivative implies that, a balanced-budget increase in τ_L shifts both of the curves in Figure 1 to the left.⁷ Some firms shift from the high productivity region to the low productivity region, increasing total output Y .⁸ There is an accompanying change in net profits. Its sign is indeterminate, but bounds can be placed on its magnitude

Lemma 6. *The change in net profits following a balanced-budget increase in τ_L satisfies*

$$-\frac{1}{s_H^*} < \frac{d\pi}{d\tau_L} < \frac{1}{s_L^*}$$

If the objective of equalization payments is to equalize access to public goods, they are successful.

Proposition 7. *A balanced-budget increase in τ_L increases government revenue in the low productivity region and decreases it in the high productivity region.*

The gap between μ and 1 is the distortion associated with this tax regime. The value of another unit of public goods measured in private goods is μ , while the cost of another unit of public goods measured in private goods is only 1. The gap is therefore the net gain from provided another unit of public goods, again measured in private goods. Equalization payments tend to equalize the distortion across the regions.

⁷An increase in τ_L forces a decrease in τ_H , so that s_H^* decreases. An increase in τ_L increases s_L^* and therefore decreases $1 - s_L^*$.

⁸Successive increases in τ_L will eventually push so many firms into region L that total output will begin to decline. However, (17) implies that $\partial Y / \partial s_i$ can only be smaller in region L if μ_i is smaller in region L , and that possibility has already been assumed away.

Proposition 8. *Assume that τ_H is always equal to $-\tau_L$. When τ_L is sufficiently near zero, an increase in τ_L causes μ to fall in the low productivity region and rise in the high productivity region.*

Furthermore, equalization payments reduce the interregional disparity in utility.

Proposition 9. *A balanced-budget increase in τ_L increases the utility of the agents in the low productivity region and reduces the utility of the agents in the high productivity region.*

Equation (17) continues to hold under unconditional transfers. It shows that the firms retain all of the gross profits in a region that optimally provides public goods; but if the firms retain all of gross profits, no public goods can be provided. It follows that unconditional transfers cannot induce a Pareto optimal allocation. However, transfers coupled with “corrective subsidies” of the sort described by Wildasin (1989) and DePater and Myers (1994) will sometimes—but not always—do so.

Suppose that government revenues are

$$\widehat{R}(s_i, \pi; \theta_i, \tau_i, \beta_i) \equiv \tau_i + \beta_i [(1 - \alpha)\theta_i(s_i)^{1-\alpha} - s_i\pi]$$

This equation differs from (18) in that it adds an additional policy parameter β_i . A value of β_i greater than 1 indicates that region i is rewarded (with additional transfers) for each dollar of revenue that it raises. The central authority can only shift revenue from one region to the other, so it must satisfy the budget constraint

$$\sum_{i=H,L} \tau_i = \sum_{i=H,L} (1 - \beta_i) [(1 - \alpha)\theta_i(s_i)^{1-\alpha} - s_i\pi]$$

Under this policy regime, (17) is replaced by

$$\pi = \frac{dY_i}{ds_i} \left[\frac{\alpha}{\mu(s_i, \pi)\beta_i} + (1 - \alpha) \right]$$

If the economy attains a Pareto optimal allocation, dY_i/ds_i is the same in both regions and $\mu(s_i, \pi)$ is equal to 1 in both regions. It follows that β_i takes the same value in each region. Letting z be each firm’s gross profits under the Pareto optimal allocation, this equation becomes

$$\pi = z \left[\frac{\alpha}{\beta} + (1 - \alpha) \right] \tag{20}$$

Government revenues rise from 0 as β rises from 1. A successful “corrective subsidy” scheme sets β to raise the desired amount of revenue, and sets τ_H and τ_L to appropriately divide the revenue between the two regions.

Corrective subsidies cannot always induce a Pareto optimal allocation. The problem is evident from (20). The share of gross profits retained by the firms is always greater than $1 - \alpha$; equivalently, the share taken by the government is always less than α . Since gross profits are a fraction $1 - \alpha$ of output, the fraction of output that can be allocated to public good provision is always less than $\alpha(1 - \alpha)$. If the agents have sufficiently strong preferences for the public good, a corrective subsidy scheme simply cannot raise the necessary revenue.

6. Conclusions

A model in which regions bargain with individual firms has been constructed, and used to analyze equalization payments. A region in which the underprovision of goods is severe is unwilling to allow firms to retain a large share of their gross profits, making firms less willing to locate there. The impact effect of transferring revenue to such a region is to reduce the severity of the underprovision, but this effect leads to an inflow of firms which have further beneficial effects. The region from which the revenue was transferred experiences the opposite effects. Since non-contingent equalization payments raise the lower utility and reduce the higher utility: they are not Pareto improving.

A. Appendix

Proof of Lemma 2. The inequality (11) implies that the residents of region i are willing to accept at least a one unit reduction in X_i to obtain a one unit increase in R_i . Suppose that, contrary to property 1, the set of firms for which $\pi_i(j)$ exceeds $\pi_{-i}(j)$ has positive measure. Region i can slightly reduce its offers to the firms in this set without losing them, causing R_i to rise by a non-negligible amount. Since only half of the net profits of these firms accrues to the residents of region i , X_i falls by only half of the increase in R_i , implying an increase in utility. The existence of a utility-improving deviation for region i implies that this profile is not an equilibrium profile. Now suppose that property 1 holds but property 2 does not. For some k_i , assume that the measure of the smaller of the two sets is at least equal to $m > 0$. Let A and B be subsets of the first and second sets, chosen so that each subset has measure m . Then region i could reduce its offers to the

firms in set A ; since almost all of these firms have matching offers from the offer region, almost all of the firms will relocate to the other region. Simultaneously, the region could raise its offers to each firm j in set B to k_i . Almost all of these firms will relocate to region i . These changes do not affect the aggregate gross profits of the firms in region i , but they do reduce the aggregate net profits of these firms, so that R_i rises by

$$\int_A \pi_i(j) dj - mk_i = \int_A (\pi_i(j) - k_i) dj > 0$$

Since the firms that leave region i do not experience a fall in net profits, while the firms that enter it experience an increase in net profits, X_i also rises. Since both X_i and R_i increase, this adjustment is a utility-improving deviation for region i , implying that the assumed profile is not an equilibrium profile. ■

Proof of Lemma 3. To show that this condition is sufficient, assume that there is no such alternative (s'_H, s'_L) and consider the effects of an alteration in region i 's offers. Reducing the offers made to some set of firms causes these firms to relocate to the other region, which is not utility-improving by assumption. Region i could also improve its offers to some set of firms. This improvement can be decomposed into two parts: (i) an arbitrarily small improvement in the offers to a set of firms that induces these firms to choose region i over region $-i$, (ii) an improvement to the offers of firms that have chosen to locate in region i . Deviations of the first type do not raise U_i by assumption. Deviations of the second type simply shift resources from X_i to R_i , and this shift could also be accomplished by making a larger transfer in the second stage. Since the second stage transfer is assumed to be optimal, this deviation does not raise U_i . The impact on utility of a deviation involving both types of changes is the sum of the impact of the individual changes and therefore cannot raise utility. To show necessity, assume that there is an alternative division of the firms satisfying (12) such that U_i is greater. If U_i would be greater if it had a smaller share of the firms, the region i has a utility-improving deviation: reducing the offers to some firms causes them to relocate, so that the region sheds the unwanted firms. Now suppose that U_i would be greater if it had a larger share of firms under the current set of offers. It can obtain the extra firms at an arbitrarily small cost by raising the offers to some set of firms by an arbitrarily small amount, and by continuity, this deviation would raise utility. Thus, equilibrium requires that each region be content with the division of the firms under the equilibrium profile. ■

Proof of Lemma 4. Each part is proved in turn.

1. If region i has no firms, X_i is positive and R_i is zero. Since the first unit of

public goods provision is arbitrarily valuable, and since both $\widehat{R}_1(0, \pi)$ and $\widehat{X}_1(0, \pi)$ are infinite, region i 's welfare initially rises as the number of firms rises from zero. Thus, the region's optimal share of the firms is not 0.

2. Let the optimal transfer in region i be $t^*(s_i, \pi; \theta_i)$. Since each region has a positive share of the firms in any equilibrium, (15) holds in each region. By the envelope theorem,

$$V_1 = u_1|_{t^*} \widehat{X}_1 + u_2|_{t^*} \widehat{R}_1$$

Optimal provision implies

$$V_1 = u_1|_{t^*} (\widehat{X}_1 + \widehat{R}_1)$$

and, using (14),

$$V_1 = u_1|_{t^*} \left(\frac{\widehat{R}}{s_i} \right)$$

Optimal provision in region i implies that the region has no tax revenue and hence no public goods provision, which is a contradiction. It follows that, contrary to assumption, public goods are underprovided. The region makes a positive transfer to the citizens only if public goods are optimally provided, so the optimal lump-sum transfer is 0. ■

Proof of Proposition 5. Define the variable

$$m_i \equiv \frac{\partial Y_i}{\partial s_i} = (1 - \alpha)\theta_i(s_i)^{-\alpha} \quad i = H, L$$

Then

$$X_i = \left(\frac{\alpha}{1 - \alpha} \right) m_i s_i + \frac{\pi}{2}$$

$$R_i = s_i(m_i - \pi)$$

It has already been shown that s_H is greater than s_L . If m_H is greater than m_L ,

$$X_H > X_L, R_H > R_L$$

$$\frac{X_H}{R_H} < \frac{X_L}{R_L}$$

Then (5) implies that the marginal rate of substitution is smaller in the high productivity region. However, (15) can be written as

$$\pi = m_i \left[\frac{\alpha}{\mu(s_i, \pi)} + (1 - \alpha) \right] \quad (21)$$

Here, the assumption that m_H is greater than m_L implies that the marginal rate of substitution is higher in the high productivity region. Since the assumption that m_H is greater than m_L leads to contradiction, it must be that m_L is greater than m_H . Then (21) implies that the marginal rate of substitution is higher in the low productivity region. Total income is greater in the high productivity region than in the low productivity region because the high productivity region has both a larger productivity factor and a larger share of the firms. If the tax revenues of the high productivity region were no greater than those of the low productivity region, it would have the higher marginal rate of substitution. Since it is known to have the lower marginal rate of substitution, its tax revenues must be greater than those of the low productivity region. Since the governments transfer no tax revenue back to the residents, consumption of both goods is higher in the high productivity region. ■

Proof of Lemma 6. A balanced budget increase in τ_L increases s_L and decreases s_H , so

$$\begin{aligned}\frac{\partial s_L^*}{\partial \tau_L} + \frac{\partial s_L^*}{\partial \pi} \frac{d\pi}{d\tau_L} &> 0 \\ -\frac{\partial s_H^*}{\partial \tau_H} + \frac{\partial s_H^*}{\partial \pi} \frac{d\pi}{d\tau_L} &> 0\end{aligned}$$

Then:

$$\frac{\partial s_H^*}{\partial \tau_H} \bigg/ \frac{\partial s_H^*}{\partial \pi} < \frac{d\pi}{d\tau_L} < - \left[\frac{\partial s_L^*}{\partial \tau_L} \bigg/ \frac{\partial s_L^*}{\partial \pi} \right] \quad (22)$$

Evaluating the derivatives of s_i^* gives

$$\frac{\partial s_i^*}{\partial \tau_i} \bigg/ \frac{\partial s_i^*}{\partial \pi} = - \left[\frac{\mu + \mu_4 \widehat{R}_1}{\mu - \mu_2 \widehat{R}_1} \right] \quad i = H, L \quad (23)$$

Now compare the absolute values of μ_2 and μ_4 . Hold s_i constant, and let a one-unit increase in π be accompanied by a s_i -unit increase in τ_i . The increase in π alone would raise μ by both decreasing \widehat{R} and increasing \widehat{X} . The accompanying increase in τ_i prevents \widehat{R} from changing, but has no effect on \widehat{X} . That is, a one-unit increase in π has a greater impact on μ than a s_i -unit increase in τ_i :

$$\mu_2 > -s_i \mu_4$$

Since \widehat{R}_1 is negative in the neighbourhood of an equilibrium,

$$\frac{\mu + \mu_4 \widehat{R}_1}{\mu - \mu_2 \widehat{R}_1} < \frac{\mu + \mu_4 \widehat{R}_1}{\mu + s_i \mu_4 \widehat{R}_1} < \frac{1}{s_i^*} \quad (24)$$

Substituting (23) and (24) into (22) gives the required inequality. ■

Proof of Proposition 7. Suppose that R_L does not rise when τ_L rises. Inspection of (18) shows that this outcome can only occur if π rises, but then (13) implies that X_L rises. The changes in X_L and R_L cause the marginal rate of substitution to rise in region L . This outcome is inconsistent with (17): the presumed increase in π pushes up the left-hand side of the equation, while the increases in s_L and $\mu(s_L, \pi)$ push down the right-hand side. This contradiction implies that, contrary to assumption, R_L rises when τ_L rises. A symmetric argument shows that R_H falls when τ_H falls, as required for a balanced budget change. ■

Proof of Proposition 8. Consider region L . Using (17) to eliminate π from \hat{X} and \hat{R} gives

$$\frac{\hat{X}}{\hat{R}} = \frac{\left(\frac{\alpha}{1-a}\right) + \left(\frac{1}{2s_L}\right)z}{1 - z + \left(\frac{\tau_L}{\theta_L}\right)(s_L)^{\alpha-1}}$$

where

$$z \equiv \frac{\alpha}{\mu(s_L, \pi)} + (1 - \alpha)$$

The direct effect of an increase in τ_L is to reduce this ratio. The increase in τ_L (accompanied by an equal but opposite change in τ_H) also induces changes in s_L and $\mu(s_L, \pi)$. It has already been shown that s_L rises, and while the rise in s_L pushes the numerator and the denominator in the same direction, it unambiguously reduces the ratio when τ_L is sufficiently near zero. If $\mu(s_L, \pi)$ does not fall, the adjustment in $\mu(s_L, \pi)$ does not raise the ratio. Then, for τ_L sufficiently near zero, the direct and indirect effects of an increase in τ_L both push the ratio higher. By (5), this change and the observation that \hat{R} rises with τ_L , imply that $\mu(s_L, \pi)$ falls. Since this finding contradicts the premise that $\mu(s_L, \pi)$ does not fall, the premise must be false: the increase in τ_L must be accompanied by a decrease in $\mu(s_L, \pi)$. A symmetric argument shows that the budget-balancing adjustment in τ_H reduces $\mu(s_H, \pi)$. ■

Proof of Proposition 9. By the envelope theorem, the effect of a balanced-budget increase in τ_L on the utility of an agent in region L is

$$\frac{dV(s_L^*, \pi; \theta_L, \tau_L)}{d\tau_L} = V_4 + V_2 \frac{d\pi}{d\tau_L} = u_2 + \frac{1}{2} (u_1 - 2s_L^* u_2) \frac{d\pi}{d\tau_L}$$

The sign of $u_1 - 2s_L^* u_2$ is uncertain because $\mu \equiv u_2/u_1$ is greater than 1 and region L contains less than half of the firms. Suppose that $u_1 - 2s_L^* u_2$ is negative. If

$d\pi/d\tau_L$ is negative, $dV/d\tau_L$ is positive. If $d\pi/d\tau_L$ is positive, Lemma 6 gives

$$\frac{dV(s_L^*, \pi; \theta_L, \tau_L)}{d\tau_L} > u_2 + \frac{1}{2}(u_1 - s_L^* u_2) \left(\frac{1}{s_L^*} \right) = \frac{u_1}{2s_L^*} > 0$$

Now suppose that $u_1 - 2s_L^* u_2$ is positive. If $d\pi/d\tau_L$ is positive, $dV/d\tau_L$ is positive. If $d\pi/d\tau_L$ is negative, Lemma 6 gives

$$\frac{dV(s_L^*, \pi; \theta_L, \tau_L)}{d\tau_L} > u_2 - \left(\frac{1}{2s_H^*} \right) (u_1 - 2s_L^* u_2) = \left(\frac{1}{2s_H^*} \right) (2u_2 - u_1)$$

which is also positive. Similarly, the effect of a balanced-budget increase in τ_L on the utility of an agent in region H is

$$\frac{dV(s_H^*, \pi; \theta_H, \tau_H)}{d\tau_L} = -V_4 + V_2 \frac{d\pi}{d\tau_L} = -u_2 + \frac{1}{2}(u_1 - 2s_H^* u_2) \frac{d\pi}{d\tau_L}$$

where $u_1 - 2s_H^* u_2$ is negative (μ is greater than 1 and region H contains more than half of the firms). If $d\pi/d\tau_L$ is positive, $dV/d\tau_L$ is negative. If $d\pi/d\tau_L$ is negative,

$$\frac{dV(s_H^*, \pi; \theta_H, \tau_H)}{d\tau_L} < -u_2 - \left(\frac{1}{2s_H^*} \right) (u_1 - 2s_H^* u_2) = -\frac{u_1}{2s_H^*}$$

which is also negative. -633 . ■

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